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THE MATHEMATICS TEACHER

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SOME HELPS AND HINDRANCES IN TEACHING MATHEMATICS IN THE SECONDARY SCHOOL.*

BY L. E. LYNDE.

It was my good fortune in school and college to come in contact with some exceptionally good teachers and some exceptionally poor ones. The contrasts were so great and the inferences so obvious that I began my work as a teacher in the confident belief that students' failures were due to inefficient teaching. It is needless to add that a decent self-respect accompanied with a little experience soon compelled me to modify my theory and to place far more responsibility on the student than he deserved. I suppose this is the experience of every teacher. With the enthusiasm of youth we tackle our job, confident that we can do it; and when we fail, we distribute the responsibility among ourselves, our pupils, and our subjects. And the responsibility ought to be so distributed, for each has its share.

The subject that we teach is not an easy one to teach or to be taught; nor is it, I believe, a difficult one unless teacher, or student, or both make it so. This is frequently done, and mathematics is proverbially known as a difficult subject. It is a common belief that a few are born with mathematical powers, and that those who are not so blessed can never hope to gain them. I don't suppose that many of us believe that. I have never come in contact with a non-mathematical mind except as I have encountered the non-logical mind. So far as my experience goes, a student who can reason in any subject can reason in mathematics; and students who *reason* in mathematics succeed in it.

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Of course, it goes without saying that to succeed a student must have the will to succeed as well as the power to do so; and that his course will be the easier if he has a desire to succeed. We all recognize the incentive that interest furnishes, and we strive to make our mathematical work as interesting as possible. We need to remember, however, that interest and achievement go hand in hand in school as well as out of it. Each depends upon the other. Neither goes far without the other. They develop simultaneously. No boy will long maintain an interest in algebra and geometry if he does poorly in them; and no boy will long lack interest in them if he does well in them. No boy *can* do well in mathematics unless he comprehends it. In no other subject is comprehension more vital. In no other subject, I sometimes think, is it less insisted upon. It is here, I am afraid, that we teachers do our poorest work. We so often encourage manipulation in place of thought. We teach our students to depend too much upon their memories and to little upon their reason. We lead them to rely altogether too much upon authority and too little upon themselves. This accounts for many of the failures and for most of the indifference toward mathematics on the part of students.

If we demand little but manipulation from them, it is unlikely that they will see anything else in mathematics. The figures, the equations, the demonstrations that a boy writes should be the expression of his own thoughts—thoughts that, if not his originally, have become his by possession. The need for this in geometry is obvious and is recognized. I believe it is even more necessary in algebra. Too many a boy gets whatever thought he has from his own written statement, *after he has written it.* What he writes is not a record of his thought. If he pauses to contemplate his written statement, he may get a thought from it. Many do not pause; few contemplate. Parrot work, whether it be in solving a problem, in demonstrating a theorem, or in solving an equation ought to be frowned on to the limit. This is particularly true in solving algebraic equations.

Many a boy solves algebraic equations by carefully following carefully stated rules with the accuracy with which an accountant adds numbers on an adding machine, and with no more comprehension of his processes. The fault I have to find is not that he does it, but that he is taught to do it. It is not his fault that he is prevented from knowing algebra. The successive equtions that a student writes in solving a given equation should be the conscious expression of new number relations, thought out by himself. They should be the visual expression of his own thoughts,—thoughts that he thinks first, and writes afterward. If they are not, then he is not using algebra, as I understand it; and in my judgment, his teacher is at fault.

How often one meets the following:

$$\frac{3x-7}{x^2-5x+6}=\frac{1}{x-3}+\frac{1}{x-2}=$$

$$3x-7=x-2+x-3.$$

The extra equality sign isn't a careless slip—that is, it usually isn't. It usually betrays utter confusion of thought, or no thought whatever. Such an occurrence provides a text for a thorough sermon, which I think a teacher ought to preach, on what an algebraic equation is and on what it is to solve one. I frequently find it necessary to remark that an equation is not a pair of scales (some textbooks to the contrary notwithstanding), and that it need not be two numbers with a pair of short horizontal lines between them. I find that it helps some boys to comprehend if I remark that "six and two make eight" is as much an equation as "$6+2=8$." The word "equals" has so many uses and senses that a definite definition of it in algebra is helpful, even to a beginner. I like to define it as meaning "represents the same number as." This definition gives clearness to the idea of a root of an equation, and emphasizes the fact that each member of the equation is a function—an idea quite essential, to a real comprehension of the solution of an equation.

But if the student writes:

$$\frac{3x-7}{x^2-5x+6}=\frac{1}{x-3}+\frac{1}{x=2},$$

$$3x-7=x-2+x-3,$$

$$x=2,$$

how often he writes totally unconscious that each successive equation he has written states a deduction from the preceding equation,—thinking perhaps that he has merely changed the dress of his original remark, and like as not led to think so by the phraseology of his text or teacher. How misleading, for example, is the statement that "adding the same number to both members of an equation does not destroy the equality." Would not the student get a better grasp of the facts if he were told that adding the same number to each member of an equation does destroy the old equality and gives a new one. And how often we mathematicians disguise a process by saying "or" when we mean "therefore," *e.g.*:

$$2x=6$$

$$\text{or}\quad x=3,$$

or by saying "and" when we mean "or" *e.g.*, in

$$x^2-5x+6=0,\qquad x=2 \text{ and } 3.$$

These seem like petty criticisms, but I believe they account for many misconceptions on the part of students.

I know it is the fashion in some quarters to laugh at the old axioms about "adding equals to equals" and "multiplying equals by equals." As specimens of English these axioms may be open to derision, but as dynamic principles for aggressive action on the student's own initiative, they have not been replaced by any modern notion with which I am familiar. It is at least true that using them students can transform equations intelligently.

If a student has solved equations mechanically, I know of nothing which helps him more to get his bearings than such simple questions as the following: If $3x+2=8$, how much is $3x+5$? It is surprising how that little question will perplex him. Had the question been "how much is $3x$," he would have answered without hesitation, for the transposing of that 2 is the established order of procedure. But the idea of wishing to get the value of $3x + 5$! Eventually he sees that since $3x + 5$ is 3 more than $3x + 2$, $3x + 5$ must be 11. (Of course some ingenious boy solves the equation for x and then computes $3x + 2$. That adds emphasis to the point of the question.) When this has been answered, I ask: If $3x + 2 = 8$, how much is $6x + 4$? $6x + 5$? $9x^2 + 12x + 4$? $\sqrt[3]{3x + 2}$? etc. Such questions bring out the relations which exist between one equation and a following one in a solution. They give a student some sense of what the following equation should be; but, best of all, they give him an idea which some students miss, and which a student ought to get early in his work, that the following equation may be *any* number relation he chooses to deduce from a given euqation. In this deduction, his first care should be whether the deduction is true, and his second care whether it serves his purpose. The axioms are his guide in the former and experience his guide in the latter.

I frequently ask a class to write out a solution thus:

If $$\frac{3x - 7}{x^2 - 5x + 6} = \frac{1}{x - 3} + \frac{1}{x - 2},$$

then $$3x - 7 = x - 2 + x - 3,$$

then $$3x - 7 = 2x - 5,$$

then $$x = 2.$$

This scheme appeals especially to those who have studied geometry. They recognize the logic at once. They realize that at any stage of the work, the next equation to be written is the equation which expresses their own deduction at the moment, and best of all, they realize—what so often isn't appreciated—the fact that the ordinary solution of an equation isn't a solution at all, but is the converse of a solution. The checking of an equation becomes to them more than a means of detecting errors; it is an integral part of the solution. (The work above doesn't show that 2 is a root. It shows that if the equation has a root, that root is 2. The check determines whether or not 2 is a root of the equation.)

I hope I am not misunderstood. The solution of equations may be taught purely as an art. A set of rules may be drawn up by following which one may determine the roots of an equation without checking. I am not attempting to argue that students can't be taught to solve equations in that way. I believe and I am trying to give utterance to the belief that this is a better way. (Of course 2 isn't a root of the above equation. One can explain away the difficulty in this example by showing how a root may be introduced into an equation by multiplying by an expression which involves the unknown, and moralizing therefrom. That is the mechanical way of meeting the situation. That to my mind is an additional reason why the deductive method of solving an equation is preferable.)

I believe that our aim in teaching mathematics should be to develop the mathematical sense of the student. The art of computation is then merely this sense applied. The mere doing of examples will, of course, enable a student, if he does enough of them, to do others like them, but it prepares him very poorly to do others of a different type. A student must know how to perform the six fundamental operations on the numbers of algebra. This requires drill, but not mere drill. The quality of the examples done counts far more than the quantity. Many texts have too many examples which students can do and do do by rote. In doing them the student applies no principle and seeks no goal. He does each like the last. His question is not "what am I seeking to do, and how shall I do it?" but "what shall I do next?" And all too frequently that question—"what shall I do next?"—is answered. I believe we give too much attention to the steps our students take and too little instruction as to where they are going. We teach them how to add fractions, but we say little or nothing as to what is meant by the sum of two fractions. We teach them how to multiply radicals but not what the product of two radicals is. We teach them to factor as though factoring were the sole purpose of algebra, but do we teach them when to factor and why? It takes some weeks each fall for me to convince some of the boys who come to me that $(a+b)(a-b)$ is not a better answer to all questions than $a^2 - b^2$. We teach students to reduce fractions by cancelling common factors in numerator and denominator, and the trouble is that is literally what they do. I am not giving utterance now to the old complaint that they cancel quantities that are not factors. I am calling attention to the fact that many students persist in writing

$$\frac{6a-2d}{4b-18d}=\frac{\cancel{2}(3a-d)}{\cancel{2}(2b-9d)}$$

instead of dividing numerator and denominator by 2. Some never reach the stage where they reduce

$$\frac{x^3-y^3}{x^2-y^2}$$

without writing

$$\frac{\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)}(x+y)}$$

Students who express themselves in that way probably haven't a very clear idea of what they are doing. It is usually another case of being so bound up in the mechanics of an operation, that the operation itself is lost sight of.

Many a student knows that $\sqrt{8}=2\sqrt{2}$ just as he knows that 2 times 3 is 6. He has memorized the fact. He can't simplify $\sqrt[5]{64}$ because that number wasn't mentioned in the book he studied. The same boy wouldn't hesitate a moment to write $\sqrt{4+a}=2+\sqrt{a}$, and there isn't the slightest reason why he should hesitate, for he used no principle in simplifying $\sqrt{8}$, and probably doesn't use principles in algebra. It may be that his introduction to the reduction of surds was an explicit as the following illustrative example given in a very good textbook:

$$\sqrt{\tfrac{2}{3}}=\sqrt{\tfrac{6}{9}}=\sqrt{\tfrac{1}{9}\cdot 6}=\tfrac{1}{3}\sqrt{6}.$$

This reveals essential steps in the process, but the only principle of surds involved in the work—in passing from the third to the fourth expression—is omitted. The one step that needs the greatest elucidation gets none. Is it any wonder that a student often gets only a fact and infers no principle?

Rules are at times a great convenience, and at times almost a necessity. But I wish that authors of texts did not feel obligated to state a rule for every process. How often a rule is stated when it should have been omitted, and how often a desirable rule is stated just when it shouldn't have been stated. An author has given a discussion, say, or has performed several illustrative examples. The class is on the point of grasping what he has done. It is ready to infer from the work done a principle for future action. It needs the experience of phrasing its own thoughts and testing them. The whole process is spoiled by the foresight of the author. He states a rule, in type so bold that the normal student sees nothing else on the page. At any rate he reads it first, and is prevented from using his own powers of inference. I can illustrate this by an instance in an exceptionally good textbook in trigonometry. The author gives an unusually complete discussion of interpolation—clear enough for any mind to comprehend. But instead of giving the student a chance to test his comprehension, the author spoils his discussion by framing a rule. He tells the student that to find a function of a given angle, he should write down the function of the next smaller angle in the table. The author then tells him how to find the correction, and adds this significant clause: "if sine or tangent, add the correction; if cosine or cotangent, subtract it." Such procedure seems to me almost criminal. If a boy has secured any comprehension of interpolation from the discussion, he doesn't need that rule. A glance at his tables will tell him whether the function is increasing or diminishing. If he does need that clause, he ought to be forced to study the discussion again, and to get a comprehension of it. But he isn't compelled to; he is told what to do, and he does it. He gets answers to four places correctly and thinks he knows all that is practical in trigonometry, and is encouraged to think so. Had he not been told what to do, he would have discovered it. He would have experienced the thrill and satisfaction of a discoverer. He would have been better equipped for future discoveries. He would have gotten more courage for the next venture and more faith in his own powers.

In solving equations by factoring, what more does a student need than appreciation of the fact that the product of several numbers can't be zero, unless one of the numbers is zero! But how many books stop there? Most of them add a rule—

giving to it as much prominence, if not more, than they give to the principle—and saying in some form that one should "*set*" each factor equal to zero. This implication of arbitrariness isn't lost on some minds. If one sets each factor equal to zero, why not set each equal to 10? Some do. I can't see that a rule here serves any useful purpose. Most students would know more mathematics if our texts contained fewer rules.

I referred some minutes ago to the increased reverence for the checking of an equation that a student has, if he comprehends the successive equations he writes in the so-called solution of the equation. There are more students than there were some years ago who realize that the letters used in algebra represent numbers. The modern insistence on the use of the check is responsible for this, I am sure. But I believe that the check has not contributed as much as it might toward increased knowledge and power in mathematics, because of the careless way in which we allow our students to check their results. I know many texts and many teachers who not only allow, but teach their students to check an equation by substituting throughout the equation the apparent value of the unknown for the unknown, and operating on the result as though "solving an equation." In checking

$$\frac{x^2}{2x^3-2}=\frac{2x}{3x^2+3x+3}-\frac{1}{6x-6}$$

for $x = -\frac{1}{5}$, they write: Check

$$\frac{\frac{1}{25}}{-\frac{2}{125}-2}=\frac{-\frac{2}{5}}{\frac{3}{25}-\frac{8}{5}+3}-\frac{1}{-\frac{6}{5}-6}$$

$$-\tfrac{5}{252}=-\tfrac{10}{63}+\tfrac{5}{36}$$

$$-5=-40+35$$

$$-5=-5.$$

This form of check may do for the expert mathematician, who knows more about indentities and reversible operations than students who are studying elementary mathematics know. It may do for him, I say, provided he is doing it as scratch work for his own use, and is making no attempt to state his thought and conclusions formally. It is almost worse than no check at all for the boy who is learning algebra. It teaches him to assume the one thing he is trying to find out—whether for this value of x the left member and the right member represent the same number. It teaches him to play with the equality sign and not to use it to make a serious statement. But worse than this, it leads many students to infer that a statement can be established by showing that truth flows from it. The first statement in the check is true, he infers, because from it he obtained a conclusion obviously true, *i.e.*, $-5=-5$. I believe that we can teach nothing more practical in mathematics than that no statement can be proved by assuming it. Obvious as that truth appears, a goodly portion of the human race needs instruction upon it. I wish that one qualification required of every voter was evidence of his appreciation of the fact that the converse of an established proposition is not necessarily true. Of course I realize that this requirement cannot be made. It would disfranchise too many voters. But I am almost ready to say that the entire time ordinarily devoted to mathematics is justified, if our students emerge from it fully realizing that the converse of an established proposition is not necessarily true and that a proposition can never be established by assuming it. Insistence on some such arrangement of the check as follows, greatly assists the student to work understandingly:

When $x=-\frac{1}{5}$,

$$\text{L. M. (left member)} = \frac{\frac{1}{25}}{-\frac{2}{125}-2} = -\frac{5}{252},$$

$$\text{R. M.} = \frac{-\frac{2}{5}}{\frac{3}{25}-\frac{3}{5}+3} - \frac{1}{-\frac{6}{5}-6} = -\frac{10}{63}+\frac{5}{36} = -\frac{5}{252}.$$

Frequently a check is so written that it doesn't involve as much bad reasoning as the earlier check. *E.g.*,

$$4+(x-3)(x-5)=15-(7-x)(2+x)$$

$$x=6.$$

Check.

$$4+(6-3)(6-5)=15-(7-6)(2+6)$$

$$4+3=15-8$$

$$7=7.$$

The fundamental error is as bad in this as in the earlier one, and in my opinion no check in this form should be accepted.

Another instance of the same bad habit occurs repeatedly in trigonometry, in establishing some of the so-called identities. To take a simple illustration:

$$\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$

$$\sin^2 x = 1 - \cos^2 x,$$

$$\sin^2 x + \cos^2 x = 1,$$

$$1 = 1. \qquad \text{Q.E.D.}$$

How often this is allowed to stand as a proof not that $1 = 1$, but that

$$\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}.$$

So long as mathematicians accept such proofs as this, I shall not initiate a referendum petition for a change in the qualifications of a voter.

It sometimes happens that students who do fairly well in abstract algebra have little success in solving problems. I find that this is sometimes due to the mistaken point of view from which they approach the problem. When a student says "I can't get an equation," I encourage him to keep trying. I assume that he knows what he is trying to do, and that he is likely soon to do it. But if he says "I can't get *the* equation," I recognize a job for myself. That boy, if his speech reveals his mind, looks upon a problem as a puzzle, in which some one has hidden an equation which it is his duty to find. He sets out looking for that equation; and if he has been unsuccessful in the past in playing this kind of hide and seek, he sets out with little expectation of finding the equation. It is some comfort and inspiration to him to be told that a problem can be solved by more than one equation; that in most problems there are many different number relations; that what he should seek is not a particular equation, but any equation involving his unknown, and not necessarily the equation in the mind of the author. This gives him hope and some courage. But if he still seeks for an equation *as such,* he is unlikely to find it. He needs to be told to think out his number relations in English, and when he has obtained an equation in English to translate it into algebra.

The equations used in solving problems necessarily deal with concrete rather than abstract numbers. I have found that a requirement that a student shall indicate the denomination of the number he has in mind in his equations is of great aid to me in interpreting his work and to him in clear thinking. *E.g.,*

$$\frac{3}{x}\text{ hrs.} - \frac{2}{x+3}\text{ hrs.} = 3\tfrac{1}{2}\text{ min.}$$

This is particularly illuminting if the conditions of the problem are such that the 3 represents miles and the x speed in miles an hour. Not the least advantage is that many half-baked ideas can be discovered in this way and put back for further cooking. For example, in the typical work-problem equation

$$\frac{1}{x} + \frac{1}{y} = \tfrac{1}{3},$$

where A and B together do a job in 3 days, the boy who doesn't comprehend is pretty sure to write:

$$\frac{1}{x}\text{ days} + \frac{1}{y}\text{ days} = \tfrac{1}{3}\text{ days},$$

or some other statement which reveals his careless thinking as the abstract equation

$$\frac{1}{x} + \frac{1}{y} = \tfrac{1}{3}$$

would not do.

Another detail which I believe is very important in its practical bearing on the solution of problems is the insistence that a student shall definitely define his unknown; that is, that he shall write not "let x = the cost," but "let x = the cost in dollars," or beads, or cents, as the case may be; not "let y = the speed," but "let y = the speed in yards a year." Experience proves that few students can afford to be less explicit, and that many profit by this attention to detail.

If then our chief concern in teaching mathematics should be, as I believe, to impart a knowledge of its principles, and a comprehension of them, it follows that the tests and examinations we give our classes should seldom, if ever, call for mere duplication of work already done. Examination questions should very enough from questions already drilled upon, to ensure that the student has gotten hold of the principle involved and can use it. Students, like the rest of the human race, do little more than is expected of them. A class is more alert, more eager to catch a truth, and the whole of the truth, if it realizes that a question will soon be asked which requires for an answer more than the repetition of what text and teacher have said. A good examination question never departs widely from questions already discussed in class, but it should seldom be identical with any of them.

This ideal is easily obtained in algebra and trigonometry. It must be modified frequently in geometry. But even here our classes can and should do better work. It is possible to insist on actual demonstration by the pupil and to get it. The demonstration of a theorem need not be a mere recitation. No student should receive a pass grade who merely recites geometry, and in my judgment, few will be candidates for a pass grade on that basis, if the ideal of grasp of principle and application of it are duly presented. I question the effectiveness of tests in geometry containing, say, three book theorems for demonstration and one question on so-called original work. Other things being equal, the ratio ought to be reversed. (Of course I do not mean to assert that a test of the first type should never be given. I give them at times myself. I am speaking of the undesirability of confining examinations to that type, or of making them predominantly of that type.) Certainly no student should be passed in geometry if he has so failed to catch its spirit and purpose, that he cannot use geometry.

Apparently we teachers of geometry do not insist that our pupils shall know enough of the facts of geometry. Every student ought to know as a fact each theorem which has been deemed important enough to be placed as a proposition in the textbook. He should know it as a fact even if he cannot prove it, and it should be at his tongue's end for statement. And he should know scores of other facts which have occurred as exercises. It has been my experience that students' inability to do so-called original work in geometry is more often due to ignorance of facts than to inability to use them. Numerical problems based on the facts of geometry not only furnish material for coördination between algebra and geometry, but they offer one of the best means for drilling and testing in geometric facts. The average text in geometry could well contain a larger number of numerical exercises. And I would place them sufficiently distant from the theorems upon which their solution depends, to permit a student to use his own initiative and judgment.

No teacher ought to be satisfied with his own work, and I can't recall that I ever met one who was. We are all eager to perfect our work by improved methods of teaching, the introduction of new material, and the elimination of old material when we are sure that such changes operate for the real benefit of the student. Some time ago the complaint arose that students in the sciences were not able to solve the equations they met there, and we teachers of mathematics were blamed for this situation, and we deserved it. Into the new texts in algebra the equations of physics and other sciences were placed; and we have drilled our students on these equations until the complaint of the science teachers isn't as loud as it was. But I wonder if we thereby have made our teaching of algebra more practical. Unless by this means we have improved our teaching of the solution of algebraic equations as such, I don't believe we have done so. A student who knows how to solve equations will solve them, whether he meets them in physics or in biology, in June or in August, whether they are expressed with x, y, and z or with M, F, and T. It is a mistake to think that algebra is made more practical by mere drill on any particular application of algebra. Some believe that time spent in teaching complex fractions, the binomial theorem, the square roots of binomial surds, etc., is wasted. It is doubtless true that some of us spend more time on these topics than we should, and that the occasions for direct application of them are few. But it by no means follows that they should be entirely omitted from our courses. Shall we teach complex fractions? The answer to that question should not depend on whether a plumber, or an engineer, a banker, or a statistician uses complex fractions. The real question before a real teacher is, does the study of complex fractions assist a student enough in getting a grasp of mathematical principles to make the study of complex fractions worth while.

There are many committees working at present to improve the teaching of mathematics. Much has been done and much will be done in elimination of work here and the introduction of new work there, in the rearrangement of material and of courses. We look for a greater diffusion of mathematical knowledge as a result of the labors of these committees; but I believe there is a greater need than this inspecting and auditing, this rejecting and renewing of material. There is, I believe, a vast difference between teaching a boy how to do mathematics, and teaching him how to do it himself. We may add special topics to our courses, or omit them; we may drill in this spot or in that; we may teach algebra one year, and geometry the next; we may teach algebra on Mondays and geometry on Tuesdays; we may teach both together on Wednesdays; we may use graphs to illustrate simultaneous equations, or simultaneous equations to illustrate graphs; we may drill our would-be engineers on engineering problems, and our would-be chemists on chemical problems—unless we teach our students to comprehend their processes, they will only be able to do things like those they have done; they will not be able to do others. They will not be able to apply mathematics. There is no substitute for reason in a subject so essentially rational as mathematics.

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